Enhancement in the dynamic response of a viscoelastic fluid flowing through a longitudinally vibrating tube

David Tsiklauri* and Igor Beresnev
Department of Geological and Atmospheric Sciences, Iowa State University, 253 Science I, Ames, Iowa 50011-3212
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We analyzed the effects of elasticity on the dynamics of fluids in porous media by studying the flow of a Maxwell fluid in a tube, which oscillates longitudinally and is subject to an oscillatory pressure gradient. The present study investigates novelties brought about in the classic Biot theory of propagation of elastic waves in a fluid-saturated porous solid by inclusion of non-Newtonian effects that are important, for example, for hydrocarbons. Using the time Fourier transform and transforming the problem into the frequency domain, we calculated (a) the dynamic permeability, and (b) the function $F(\kappa)$ that measures the deviation from Poiseuille flow friction as a function of frequency parameter κ. This provides a more complete theory of the flow of Maxwell fluid through a longitudinally oscillating cylindrical tube with an oscillating pressure gradient, which has important practical applications. This study clearly shows the transition from a dissipative regime to an elastic regime in which sharp enhancements (resonances) of the flow are found.

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I. INTRODUCTION

A quantitative theory of propagation of elastic waves in a fluid-saturated porous solid was formulated in classic papers by Biot [1]. One of the major findings of Biot's work was that there is a breakdown in Poisseuille flow above a certain characteristic frequency specific to the fluid-saturated porous material. Biot theoretically studied this phenomenon by considering the flow of a viscous fluid in a tube with longitudinally oscillating walls under an oscillatory pressure gradient. Apart from its fundamental interest, the investigation of the dynamics of fluid in porous media, under an oscillatory pressure gradient and oscillating pore walls, is of prime importance for the recently emerged technology of acoustic stimulation of oil reservoirs [2]. For example, it is known that natural pressure in an oil reservoir generally yields no more than approximately 10% oil recovery. The residual oil is difficult to produce due to its naturally low mobility, and the enhanced oil recovery operations are used to increase production. It was experimentally proven that there is a substantial increase in the net fluid flow through porous space if the latter is treated with elastic waves. However, there is a fundamental lack of understanding of the physical mechanisms of fluid mobilization in saturated rock through the effect of elastic waves; the theory of such mobilization virtually does not exist. Biot's theory can be used to describe the interaction of a fluid-saturated solid with the sound for a classic Newtonian fluid; however, oil and other hydrocarbons exhibit significant non-Newtonian behavior [3]. The aim of this paper is therefore to incorporate non-Newtonian effects into the classical study of Biot [1].

Recently, del Rio, Lopez de Haro, and Whitaker [4] presented a study of enhancement in the dynamic response of a viscoelastic (Maxwell) fluid flowing in a stationary (nonoscillating) tube under the effect of an oscillatory pressure gradient. We combine this theory with the effect of acoustic oscillations of the walls of a tube introduced by Biot [1], providing a complete description of the interaction of Maxwell fluid, filling the pores, with acoustic waves.

Finally, in order to emphasize that the concept of dynamic permeability is an adequate way to describe the phenomenon, we note that this concept has been widely used before [5]. In the Sec. II, we formulate our model, and Sec. III concludes with a discussion of the results.

II. MODEL

In this section we present our model of a Maxwell fluid flowing in a cylindrical tube whose walls are oscillating longitudinally and the fluid is subject to an oscillatory pressure gradient. We give analytical solutions of the problem in the frequency domain.

The governing equations of the problem consist of the continuity equation for the incompressible fluid,

$$\vec{\nabla} \cdot \vec{v} = 0, \tag{1}$$

and the linearized momentum equation,

$$\rho \frac{\partial \vec{v}}{\partial t} = -\vec{\nabla} p - \vec{\nabla} \tilde{\tau}. \tag{2}$$

Here $\vec{v}$, p, and ρ denote velocity, pressure, and mass density of the fluid, whereas $\tilde{\tau}$ represents the viscous stress tensor. We describe the viscoelatic effects of the fluid using Maxwell's model, which assumes that

$$t_m \frac{\partial \tilde{\tau}}{\partial t} = -\eta \vec{\nabla} \vec{v} - \tilde{\tau}, \tag{3}$$

where η is the viscosity coefficient, and t_m is the relaxation time.

Now let u be the velocity of the wall of the tube which oscillates in time as $e^{-i\omega t}$, where ω is the angular frequency.

*Email address: dtsiklau@iastate.edu beresnev@iastate.edu

The flow of fluid in a cylindrical tube with longitudinally oscillating walls can be described by a single component of the velocity, namely, its z component v_z (the z axis is along the centerline of the tube). We use the cylindrical coordinate system (r,ϕ,z) in the treatment of the problem. We introduce the relative velocity U_1 as $U_1=v_z-u$. Thus, assuming that all physical quantities vary in time as $e^{-i\omega t}$, we arrive at the following master equation for U_1:

$$\nabla^2 U_1+\frac{\omega^2 t_m+i\omega}{\nu}U_1=-\frac{X}{\nu}(1-i\omega t_m). \tag{4}$$

Here we have introduced the notations

$$X=-\left(\nabla p+\rho\frac{\partial u}{\partial t}\right),$$

which is a sum of the applied pressure gradient and force exerted on the fluid from the oscillating wall of the tube and ν, which is $\nu=\eta/\rho$. Note that a no-slip boundary condition at the wall is assumed.

The solution of Eq. (4) can be found to be [1]

$$U_1=-\frac{X}{i\omega}+CJ_0(\beta r),$$

where J_0 is the Bessel function, and $\beta=\sqrt{(\omega^2 t_m+i\omega)/\nu}$.

Applying a no-slip boundary condition $U_1(a)=0$ at the wall of the tube, where a is its radius, we finally obtain

$$\begin{aligned}U_1(r)&=-\frac{X}{i\omega}\left[1-\frac{J_0(\beta r)}{J_0(\beta a)}\right]\\&=-\frac{Xa^2(1-i\omega t_m)}{\nu}\frac{1}{(\beta a)^2}\left[1-\frac{J_0(\beta r)}{J_0(\beta a)}\right].\end{aligned} \tag{5}$$

Defining the cross-section-averaged velocity as

$$\bar{U}_1=\frac{2}{a^2}\int_0^a U_1(r)r\,dr,$$

we obtain

$$\bar{U}_1=-\frac{Xa^2(1-i\omega t_m)}{\nu}\frac{1}{(\beta a)^2}\left[1-\frac{2J_1(\beta a)}{(\beta a)J_0(\beta a)}\right]\equiv K(\omega)X. \tag{6}$$

Here $K(\omega)$ is the dynamic permeability [4] that describes the frequency dependent response of the tube to the applied total force on the fluid. A simple comparison reveals that Eq. (6) closely resembles Eq. (6) of Ref. [4], with the only difference being that we have X in place of their $\partial p/\partial z$. Note that in the case of stationary tube walls $[\rho(\partial u/\partial t)\rightarrow 0]$ Eq. (6) coincides exactly with Eq. (6) of Ref. [4]. A simple calculation (applying L'Hospital rule for the 0/0 uncertainty) shows that $\lim_{\omega\rightarrow 0}K(\omega)=a^2/(8\nu)$. Thus, following Ref. [4], we will introduce the dimensionless dynamic permeability as $K^*(\omega)=K(\omega)/K(0)$, which will be used later (see Fig. 7). Note that we were easily able to reproduce Fig. 1 of Ref. [4], confirming the existence of sharp resonances of $K^*(\omega)$ in the elastic regime (see below for the definition of this regime).

Following the work of Biot [1], we calculate the stress at the wall τ:

$$\tau=-\frac{\eta}{1-i\omega t_m}\left(\frac{\partial U_1(r)}{\partial r}\right)_{r=a}=\frac{\eta\beta X}{i\omega(1-i\omega t_m)}\frac{J_1(\beta a)}{J_0(\beta a)}. \tag{7}$$

Note that when $t_m\rightarrow 0$, this expression obviously coincides with the corresponding Newtonian form.

The total friction force is $2\pi a\tau$. Following Biot, we calculate the ratio of the total friction force to the average velocity, i.e.,

$$\frac{2\pi a\tau}{\bar{U}_1}=-\frac{2\pi\eta(\beta a)[J_1(\beta a)/J_0(\beta a)]}{(1-i\omega t_m)}\times\left[1-\frac{2J_1(\beta a)}{(\beta a)J_0(\beta a)}\right]^{-1}. \tag{8}$$

A simple analysis reveals that

$$\lim_{\omega\rightarrow 0}\frac{2\pi a\tau}{\bar{U}_1}=8\pi\eta,$$

which corresponds to the limiting case of the Poiseuille flow. Following Biot [1], we also introduce a function $F(\kappa)$, with $\kappa=a\sqrt{\omega/\nu}$, in the following manner:

$$\frac{2\pi a\tau}{\bar{U}_1}=8\pi\eta F(\kappa);$$

thus

$$F(\kappa)=-\frac{1}{4}\frac{\kappa\sqrt{i+\kappa^2/\alpha}[J_1(\kappa\sqrt{i+\kappa^2/\alpha})/J_0(\kappa\sqrt{i+\kappa^2/\alpha})]}{(1-i\kappa^2/\alpha)}\times\left[1-\frac{2J_1(\kappa\sqrt{i+\kappa^2/\alpha})}{\kappa\sqrt{i+\kappa^2/\alpha}J_0(\kappa\sqrt{i+\kappa^2/\alpha})}\right]^{-1}. \tag{9}$$

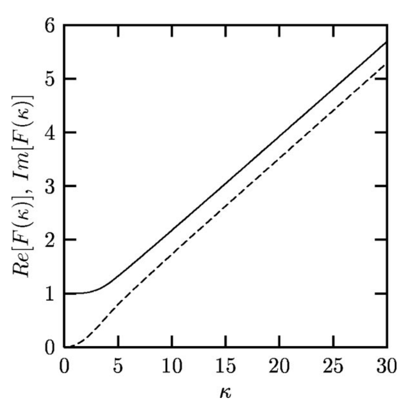

FIG. 1. Behavior of Re$[F(\kappa)]$ (solid line) and Im$[F(\kappa)]$ (dashed line) as functions of κ according to Eq. (9). Here $\alpha=\infty$.

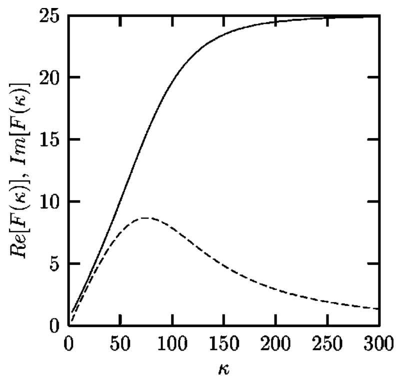

FIG. 2. Same as in Fig. 1, but for $\alpha = 10^4$.

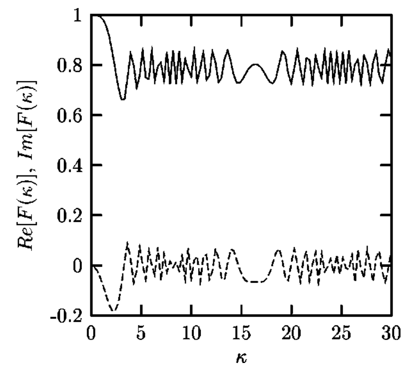

FIG. 4. Same as in Fig. 1, but for $\alpha = 10.0$.

Note that $F(\kappa)$ measures the deviation from the Poiseuille flow friction as a function of the frequency parameter κ, as introduced by Biot [1].

In Eq. (9), α denotes the Deborah number [4], which is defined as the ratio of the characteristic time of viscous effects $t_v = a^2/\nu$ to the relaxation time t_m, i.e. $\alpha = t_v/t_m = a^2/(\nu t_m)$. As noted in Ref. [4], the value of the parameter α determines in which regime the system resides. Beyond a certain critical value ($\alpha_c = 11.64$), the system is dissipative, and viscous effects dominate. On the other hand, for small α ($\alpha < \alpha_c$) the system exhibits a viscoelastic behavior which we call the elastic regime.

Note that the Newtonian flow regime can be easily recovered from Eq. (9) by putting $\alpha \to \infty$. We plot this limiting case in Fig. 1. As can be seen from the figure, both $\mathrm{Re}[F(\kappa)]$ and $\mathrm{Im}[F(\kappa)]$ coincide exactly with the Newtonian limiting case studied in Biot's work (see Fig. 4 in Ref. [1]). This graph demonstrates a breakdown in the Poiseuille flow as the frequency increases (recall that $\kappa \propto \sqrt{\omega}$). In all our calculations we have used polynomial expansions of J_0 and J_1, with an absolute error not exceeding $10^{-6}\%$. Thus our calculation results are accurate to this order.

A finite-but-large α regime is shown in the next two figures. Figure 2 corresponds to the case when $\alpha = 10^4$. We see in Fig. 2 that the real and imaginary parts of $F(\kappa)$ start to deviate from the Newtonian fluid behavior at *large* frequencies. In Fig. 3, solutions correspond to the case when $\alpha = 100.0$; thus we see how viscoelastic effects already become pronounced at *low* frequencies.

Figure 4 presents the behavior of $\mathrm{Re}[F(\kappa)]$ and $\mathrm{Im}[F(\kappa)]$ when $\alpha = 10.0$. As can be seen from the graph, sharp resonances appear on the curves. This feature can be explained by the fact that in this case α is less than α_c, which means that the system switches from a dissipative (viscous) regime to an elastic one.

Figure 5 shows the solutions when $\alpha = 1.0$. In this case we see more irregular behaviors of $\mathrm{Re}[F(\kappa)]$ and $\mathrm{Im}[F(\kappa)]$ with a number of irregular spikes.

The extreme non-Newtonian (elastic) regime is studied in Fig. 6, where we plot the solutions for the case when $\alpha = 10^{-3}$. In this case a notable change is that there are fewer but more pronounced spikes. $\mathrm{Re}[F(\kappa)]$ is close to zero for most of the frequencies, and only at certain frequencies do we see sharp resonances. In this regime the system acts as a *window* for these frequencies.

Another noteworthy observation is that, on the one hand, as long as $\alpha > \alpha_c = 11.64$ (Figs. 2 and 3), $\mathrm{Re}[F(\kappa)]$ is always greater than its initial value, i.e., $\mathrm{Re}[F(\kappa)] > 1$, and for large frequencies it reaches a certain asymptotic value. On the other hand, when $\alpha < \alpha_c$ (Figs. 4–6), we observe an overall decrease in $\mathrm{Re}[F(\kappa)]$ with the increase in frequency, i.e., $\mathrm{Re}[F(\kappa)] < 1$ for all κ's.

In Fig. 7 we also study the behavior of the dimensionless dynamic permeability $K^*(\omega_*)$ as a function of ω_*, the dimensionless frequency defined as $\omega_* = t_m\omega$, for the case when $\alpha = 0.1$. Since $\alpha < \alpha_c$, we observe sharp resonances in the dynamic permeability at certain frequencies, which can be explained by the non-Newtonian behavior of the fluid.

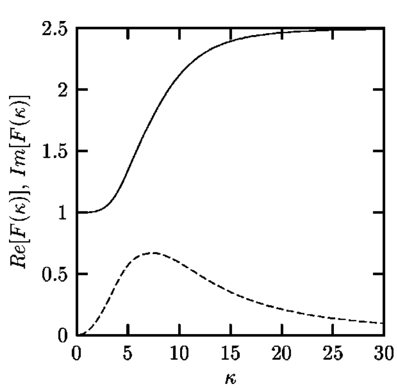

FIG. 3. Same as in Fig. 1, but for $\alpha = 100.0$.

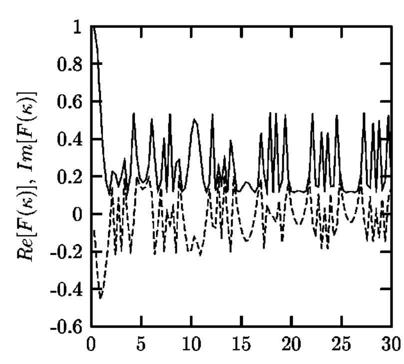

FIG. 5. Same as in Fig. 1, but for $\alpha = 1.0$.

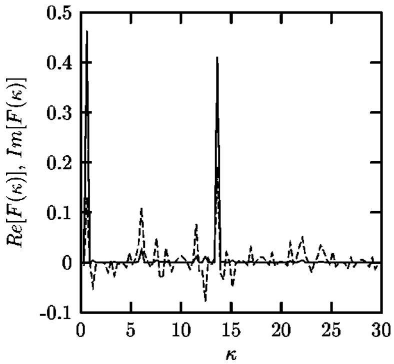

FIG. 6. Same as in Fig. 1, but for $\alpha = 10^{-3}$.

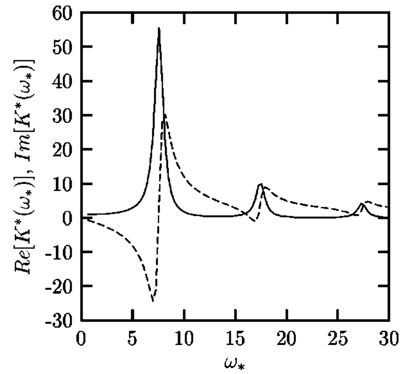

FIG. 7. Behavior of $\mathrm{Re}[K^*(\omega_*)] = \mathrm{Re}[K(\omega_*)/K(0)]$ (solid line) and $\mathrm{Im}[K^*(\omega_*)] = \mathrm{Im}[K(\omega_*)/K(0)]$ (dashed line), as function of ω_* according to Eq. (6). Here $\alpha = 0.1$.

III. DISCUSSION

In this paper we have studied non-Newtonian effects on the dynamics of fluids in porous media by calculating a flow of Maxwell fluid in a tube, which oscillates longitudinally and is subject to an oscillatory pressure gradient. The present study investigates modifications of the classic Biot theory [1] of propagation of elastic waves in a fluid-saturated porous solid by inclusion of non-Newtonian effects. We have used time Fourier transform, and converted the problem to the frequency domain. We have calculated the dynamic permeability, thus modifying the work of del Rio, Lopez de Haro, and Whitaker [4] by inclusion of the effect of longitudinally oscillating tube walls. We investigated how the function $F(\kappa)$, which measures the deviation from Poiseuille flow friction as a function of the frequency parameter κ, is modified by non-Newtonian effects. Our work thus provides a combined theory of non-Newtonian flow in a longitudinally oscillating tube, which constitutes the basis for a realistic model of the effects of elastic waves in a fluid-saturated porous space. The present analysis clearly demonstrates the existence of a transition from a dissipative regime to an elastic regime (as α decreases), in which sharp enhancements of flow (resonances) occur. The importance of the current work is twofold. (a) We studied modifications brought about by non-Newtonian effects in Biot's theory. The investigation of the function $F(\kappa)$ is important for a number of applications, since $F(\kappa)$ uniquely determines the response of a realistic fluid-saturated porous medium to the elastic waves. Thus a determination of $F(\kappa)$ for non-Newtonian (Maxwell) fluid is necessary to guide e.g., oil-field exploration applications. (b) As we have seen from Fig. 7, non-Newtonian effects cause substantial enhancements in the dynamic permeability. We were not able to determine in the literature what value of α a natural crude oil would have. However, as can be seen from Fig. 7 for $\alpha = 0.1$, we can obtain an increase in permeability of up to 60 times at certain resonant frequencies. Lower α's yield even more drastic enhancements. At any rate, we obtained an analytical expression for $F(\kappa)$ [Eq. (9)], which can provide the behavior of this function for any given α.

In Sec. I, the practical impact of the possibility of acoustic stimulation of oil reservoirs was outlined. This result clearly demonstrates that, in crude oil, which can be modeled as a Maxwell fluid, there are certain resonant frequencies at which oil production can be increased significantly if the well is irradiated with elastic waves at these frequencies.

ACKNOWLEDGMENTS

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